**Hypothesis Testing for Population Variance**

* Consider population ~ X ~ N(μ, σ)
  + Test H0: σ = σ0 vs. σ ≠ σ0
  + What is the test statistic?
* Chi-square distribution
  + Let X1 … Xn be independent r. v. that ~ N(0, 1)
  + Then S = X12 + … + Xn2 has a X2-distribution with degrees of freedom = n
    - i.e. S ~ X2n
    - i.e. if r. v. Y ~ N(0, 1), Y2 ~ X21
  + For X ~ X2, x ≥ 0
  + E(X) = n
  + Var(X) = 2n
  + X1 + X2 ~ X2n1 + n2 for independent distributions X1, X2
  + Pivotal U = (n – 1)S2/σ2 ~ X2n-1
  + (1 − α)100% CI for σ2:
    - , degrees of freedom = n – 1
  + Ex: n = 50; S2 = 25
    - Find 90% CI for σ
    - D.f. = 49 → approx. to 50
    - i.e. U ~ X250
    - Select a, b such that P(a ≤ U ≤ b) = 0.90
      * P(a ≤ U ≤ b) = P(U ≥ a) – P(U > b)
      * Find a such that P(U ≥ a) = 0.95 & b such that P(U > b) = 0.05
      * From table: a = 34.8, b = 67.5
      * ∴ CI is [4.26, 5.93]